ELSEVIER

Economics Letters 49 (1995) 175–179

economics letters

The Green Golden Rule

Graciela Chichilnisky[a], Geoffrey Heal[b,*], Andrea Beltratti[c]

[a]Columbia University, New York, NY 10027, USA
[b]Columbia Business School, Columbia University, New York, NY 10027, USA
[c]Istituto Prato, Universita di Torino, Corso Unione Sovietica 218 bis, Torino, 10134, Italy

Received 2 August 1994; accepted 8 November 1994

Abstract

We introduce a growth model with environmental assets as a source of utility and an input to consumption and production. In this model we develop the Green Golden Rule, a generalization of the golden rule of neoclassical growth theory. We apply this to the case where the object is the maximization of long-run or limiting utility rather than long-run consumption.

Keywords: Sustainable; Resources; Environment; Growth

JEL classification: D90

1. Introduction

The two key factors common to most definitions of 'sustainable growth' are respect for resource limitations and emphasis on equity among generations. Both elements have been analyzed previously: see the Bariloche model (Herrera et al., 1976), Chichilnisky (1977) and Heal (1993) (and references contained therein) for the importance of resource constraints and Ramsey (1928), Solow (1974), the Brundtland Report (WCED, 1987), Chichilnisky (1993) and Heal (1993) (and references contained therein) for intergenerational equity. So the issues raised by sustainability are not new, but there seems, nevertheless, to be dissatisfaction with the earlier answers, probably mainly in the area of equity.

This literature on intergenerational equity is largely built around the discounted utilitarian approach to defining an optimal path, and many authors have expressed reservations about the balance that strikes between present and future, see Heal (1985, 1993). Discounted utilitarianism has proven particularly controversial with non-economists concerned with

* Correspondence to: Geoffrey Heal, 101 Uris Hall, Graduate School of Business, Columbia University, New York, NY 10027, USA.

SSDI 0165-1765(95)00662-1

environmental valuations; for example Cline (1992) has argued for the use of a zero discount rate in the context of global warming. So part of our concern about the future is not captured by discounted utilitarianism. It is this that is driving an interest in formalizing the concept of sustainability. An axiomatic approach to sustainability, and a comprehensive review of alternative approaches, can be found in Chichilnisky (1993).

In this paper we introduce and analyze the 'Green Golden Rule' (GGR), an extension to the environmental field of the Meade–Phelps–Robinson concept of the Golden Rule of Economic Growth. A connection with sustainability should not be surprising. Phelps (1961) described the golden rule as the growth path that gives the highest indefinitely maintainable level of consumption per head. Clearly, there is an implicit concept of sustainability here: the Golden Rule path is the best sustainable path. Our GGR gives the highest indefinitely maintainable level of instantaneous utility, in a framework where environmental goods are valued in their own rights.

The paper is organized as follows: in Section 2 we consider a general model of growth with physical capital and a stock of environmental resource. In Section 3 we derive the GGR from the maximization of long-run utility; in Section 4 we characterize the general model with specific functions (log-linear utility and a logistic reproduction function for the stock of natural resource) in order to provide an example. Section 5 offers some concluding comments.

2. A model of growth with environment

The economic model with which we work is an extension of Dasgupta and Heal (1974). We assume that the resource is a source of utility, which is given by the strictly concave function $u(C_t, A_t)$ defined on consumption C_t and on the stock of an environmental good A_t. Production occurs according to the linear homogeneous production function $F(K_t, A_t)$ where K_t is the stock of produced capital at time t. Capital accumulation is described by

$$\dot{K}_t = F(K_t, A_t) - C_t . \tag{1}$$

We also assume that A can renew itself: the rate of renewal is given by the function $R(A)$, satisfying $R(0) = 0$. However, the act of consuming output may deplete the environment, so that the net rate of change of the stock of the environment is

$$\dot{A}_t = -C_t + R(A_t), \quad \alpha \geq 0 . \tag{2}$$

We assume that the renewal function R is bounded above, may exhibit a threshold effect and may be decreasing above a certain level of A.

3. The Green Golden Rule

In considering the case in which society is only concerned with the long-run values of consumption and environment, we seek a path maximizing $\lim_{t\to\infty} u(C_t, A_t)$. The solution is characterized in the following proposition.

Proposition 1. There exist values (A^*, K^*, C^*), characterized by $u'_A/u'_C = -R'$, such that

$$\lim_{t\to\infty} \{A_t, K_t, C_t\} = \{A^*, K^*, C^*\}$$

is a necessary and sufficient condition for a feasible path $\{A_t, K_t, C_t\}\forall t$ to be a solution of the problem 'maximize $\lim_{t\to\infty} u(C_t, A_t)$ over all feasible paths'.

Proof. As indefinitely maintainable values of C and A satisfy $R(A) = C$, this means that the problem

$$\max \lim_{t\to\infty} u(C_t, A_t)\,, \quad \text{over feasible paths}\,,$$

reduces to

$$\max u(C, A)\,, \quad \text{subject to } R(A) = C\,. \tag{3}$$

The stock of capital is not a concern in this situation because any stock of capital can be accumulated given a sufficiently long period of time. The set of $\{C, A\}$ pairs satisfying the constraint in (3) is compact, so this problem is well-defined. The maximum is characterized by the first-order condition:

$$\frac{u'_A}{u'_C} = -R'\,. \tag{4}$$

This completes the proof. □

The solution given in the proposition amounts to equality between the marginal rate of transformation and the marginal rate of substitution between consumption and environment across steady states. We term the configuration defined by (4) the 'Green Golden Rule', in reference to its relationship to the original Meade–Phelps–Robinson 'Golden Rule of Economic Growth'. While the latter characterizes the greatest indefinitely maintainable consumption level, the former (4) characterizes the highest indefinitely maintainable utility level. It is a generalization of the earlier concept, and has points in common with Brock's (1977) 'Polluted Golden Age', although he models a pollution stock rather than an environmental asset.

It is interesting to note that the GGR is the desired long-run configuration of the economy when the objective is the maximization of any purely finitely additive measure $f(u(C_t, A_t))$, where f may be equal to $\lim_{t\to\infty}$, $\lim_{T\to\infty}\inf_{t>T}$, or long-run average $\lim_{N\to\infty}\lim_{M\to\infty} 1/M \sum_{t=N}^{t=N+M} u(C_t, A_t)$. Chichilnisky (1993) has proposed a criterion for ranking utility streams, which places weight on the long-run properties of the sequence as well as on its properties over finite horizons. She derives this from representing the ranking of intertemporal utility sequences as a social choice problem, and shows that, given non-dictatorship and continuity axioms, utility sequences are ranked by $\int u(C_t, A_t)\,d\mu + f(u(C, A))$, where μ is countably additive and f is purely finitely additive. Similar considerations arise in game theory, where players are concerned with the long-run outcome of the game; see, for example, Dutta (1991).

4. A specific example

We consider here an example that ignores capital, as including the latter would not change the solution to maximization of long-run utility, but would significantly complicate the solution to the discounted utilitarian criterion. The utilitarian optimization problem is

$$\max \int_0^\infty (\ln C_t + \gamma \ln A_t)\, e^{-\delta t}\, dt ,$$

subject to the dynamic equation:

$$\dot{A}_t = -C_t + rA_t - \frac{rA_t^2}{A^S} ,$$

and to initial conditions. In the logistic reproduction function, A^S is the carrying capacity of the environment; $A^S/2$ is the stock corresponding to the maximum sustainable yield (MSY). The steady state of the stock of the resource implied by the necessary conditions is (see Beltratti et al., 1994, for a more thorough analysis):

$$A^D = \frac{A^S(\gamma r - \delta + r)}{2r + \gamma r} .$$

The relationship between such a solution and the one providing the MSY is seen more easily when $\gamma = 0$ (no direct utility of the environment), in which case the steady-state level of the environment in the discounted utilitarian case is clearly lower than $A^S/2$, corresponding to the MSY. When $\gamma > 0$ (in fact when $\gamma > 2\delta/r$) one can have an optimal stock of resource even larger than $A^S/2$ due to the direct effect on utility. We can characterize the GGR by maximizing instantaneous utility to obtain:

$$A^{GGR} = A^S\left(\frac{1+\gamma}{2+\gamma}\right) = \frac{2(A^S/2) + \gamma A^S}{2+\gamma} .$$

This is, in general, larger than the value at the utilitarian stationary solution, being:

$$A^D = A^{GGR} - \frac{\delta A^S}{r(2+\gamma)} ,$$

although the two stationary states converge as the discount rate is reduced to zero. The solution can be interpreted as a weighted average between A^S, the level that maximizes the direct effect of the stock on utility, keeping consumption constant, and $A^S/2$, the level that maximizes consumption.

5. Conclusions

The paper has analyzed a model in which utility is provided both by consumption and by a stock of environmental resource, and where the former depletes the latter. The solution to the discounted utilitarian criterion has shown the importance of the rate of time preference in discriminating against future generations, while the solution to the problem of maximizing long-run utility has pointed out the existence of the Green Golden Rule, an extension of Phelps' golden rule. In connecting such a solution to the concept of sustainability, it has been observed that the stock that maximizes utility in the long run is larger than the one corresponding to the maximum sustainable yield.

An important task for future research lies in the analysis of criteria that combine maximization of discounted utility with elements related with the long run. Such criteria are developed by Chichilnisky (1993), and a first exploration of their use in various models with physical capital and environmental resources is contained in Beltratti et al. (1994).

Acknowledgments

This work was funded in part by grants from the Fondazione ENI Enrico Mattei and NSF grants 92 16028 and 91 10460. An earlier version was presented at the OECD conference on Sustainable Economic Development, Paris, May 24–25, 1993. We are grateful to participants for valuable comments.

References

Beltratti, A., G. Chichilnisky and G. Heal, 1994, The environment and the long run: A comparison of different criteria, Ricerche Economiche 48, 319–340.

Brock, W., 1977, A polluted golden age, in: V.L. Smith, ed., Economics of natural and environmental resources (Gordon and Breach, New York) 441–461.

Chichilnisky, G., 1977, Economic development and efficiency criteria in the satisfaction of basic needs, Applied Mathematical Modeling, 1.

Chichilnisky, G., 1993, What is sustainable development?, Paper presented at the 1993 workshop of the Stanford Institute for Theoretical Economics.

Cline, W.R., 1992, The economics of global warming, Institute for International Economics, Washington DC.

Dasgupta, P.S. and G.M. Heal, 1974, The optimal depletion of exhaustible resources, Review of Economic Studies, Symposium on the Economics of Exhaustible Resources, 3–28.

Dutta, P., 1991, What do discounted optima converge to? A theory of discount rate asymptotics in economic models, Journal of Economic Theory 55, no. 1, 64–94.

Heal, G.M., 1985, Depletion and discounting: A classical issue in the economics of exhaustible resources, American Mathematical Society, Proceedings of Symposia in Applied Mathematics 32, 33–43.

Heal, G.M., 1993, The optimal use of exhaustible resources, in: A.V. Kneese and J.L. Sweeney, eds., Handbook of natural resource and energy economics (North-Holland, Amsterdam, New York and Oxford), 855–879.

Herrera, A.O., H.D. Scolnik, G. Chichilnisky et al., 1976, Catastrophe or new society, International Development Research Centre, Ottawa, Canada.

Phelps, E.S., 1961, The golden rule of accumulation: A fable for growthmen, American Economic Review 51, 638–643.

Ramsey, F., 1928, A mathematical theory of saving, Economic Journal 38, 543–559.

Solow, R.M., 1974, Intergenerational equity and exhaustible resources, Review of Economic Studies, Symposium on the Economics of Exhaustible Resources, 29–45.

World Commission on Environment and Development, 1987, Our common future (The Brundtland Report) (Oxford University Press, Oxford).